

Standard Model of Particle Physics:

In order to calculate  $g_*$ , we need to know the number of degrees of freedom that are relativistic at a given temperature and are in thermal equilibrium.

Let us discuss the "Standard Model of Particle Physics" that describes the known fundamental particles and (except for gravity) their interactions<sup>^</sup> and is in very good agreement with experiments.

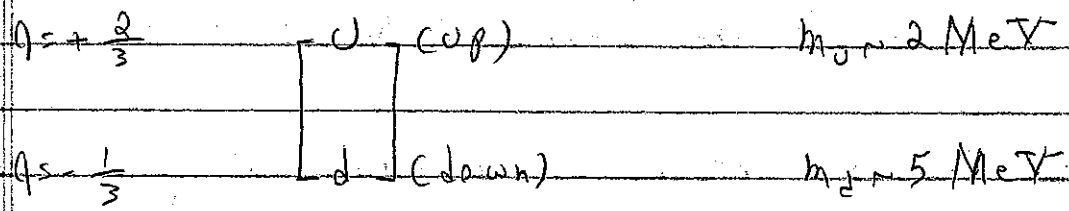
The fundamental particles come in three groups according to the Standard Model:

- 1 - Building blocks of matter; these are spin  $\frac{1}{2}$  fermions.
- 2 - Mediators of the forces; these are spin-1 bosons.
- 3 - Higgs field; it is a spin-0 boson.

We now describe each of these groups in more detail

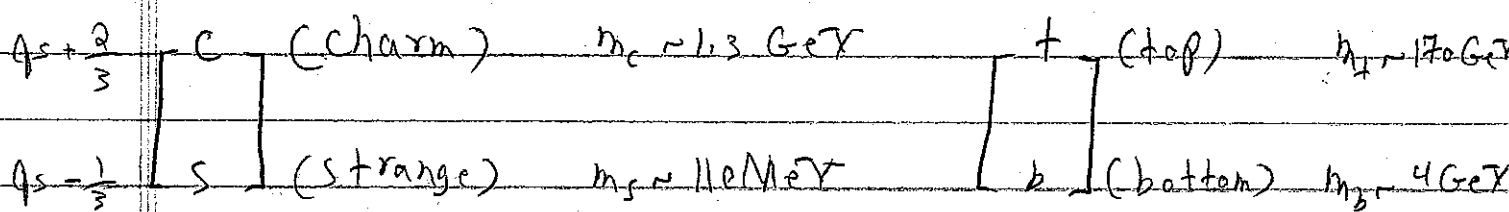
1. Building blocks of matter come in two types:

(a) Quarks. These are fermions that have all the three (electromagnetic, weak, strong) interactions. The nucleons (protons and neutrons) are bound states of two quarks;



"q" denotes the electric charge of the quarks. Proton is a udd bound state, while neutron is a udd bound state. The quarks are bound by the strong force.

There are two heavier copies of quarks with the same charge as u, d respectively:



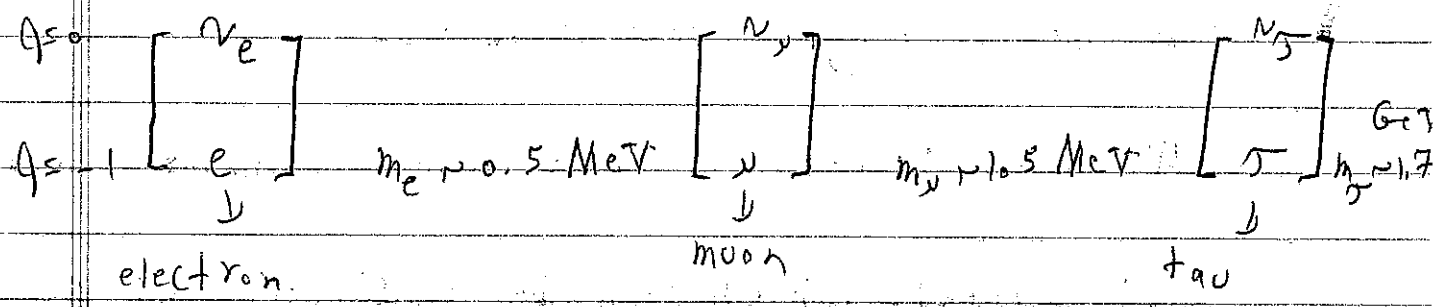
We do not see free, isolated quarks at low energies because they are bound by the strong force (which becomes strong - at low energies).

All quarks come in three colors (namely red, blue and green), which can be considered as quantum charges related to strong interactions (similar to electric charge and electromagnetic interaction).

Also note that quarks are spin- $\frac{1}{2}$  fermions. This represents four degrees of freedom (spin-up and spin-down particle and antiparticle).

(b) Leptons. These are fermions that have only weak and electromagnetic interactions. The leptons are either charged ( $q = -1$ ) or neutral ( $q = 0$ ). The lightest charged lepton is the electron that

Via electromagnetic interactions binds to nuclei and form atoms. The leptons also come in three copies (called family or flavor)



The neutrinos associated with the charged leptons are called electron neutrino, muon neutrino and tau neutrino.

From atmospheric neutrino oscillations, there exists a mass scale of  $\approx 0.05 \text{ eV}$  for the difference of neutrino masses. The combined limits from lab and cosmology puts an upper bound  $\leq 0.5 \text{ eV}$  on the sum of the mass of three neutrinos.

The charged leptons each represent four degrees of freedom. But each neutrino represents two

degrees of freedom. This is what nature seems to have dictated. A naive (but not quite correct) way to understand it is that since neutrinos are neutral particles, then particle and antiparticle can be the same (unlike the case of quarks or charged leptons). This will leave us with spin-up and spin-down states, that count for two degrees of freedom.

2 - Mediators of force are spin-1 particles. The most famous one is photon ( $\gamma$ ) that is the messenger of electromagnetic interactions:

$$g = 0 \quad \gamma \quad m_\gamma = 0$$

Since photon is massless ( $m_\gamma = 0$  compatible with all experimental bounds), then it has two degrees

of freedom (the transverse polarizations).

Next we have three spin-1 particles that are messengers of the weak interactions:

$Q=0$        $Z$        $m_Z \sim 90 \text{ GeV}$

$Q=\pm 1$        $W^\pm$        $m_{W^\pm} \sim 80 \text{ GeV}$

The fact that  $Z, W^\pm$  are massive explains why the weak interactions are short range; the exchange of  $Z, W^\pm$  gives rise to Yukawa potentials (which are exponentially suppressed in distance) unlike the photon exchange that results in a Coulomb potential with infinite range.

Since  $Z, W^\pm$  (called weak gauge bosons) are massive each of them represents three degrees of freedom: two transverse and one longitudinal polarizations.

Finally, we have the messengers of strong interactions<sup>on</sup>

that are called gluons (denoted by "G"). There are 8 gluons, and they are massless. Hence each gluon represents two degrees of freedom (like the photon). However, at low energies we do not see free, isolated gluons because they are bound by strong interactions:

$$g_s = 0 \quad G \quad m_G = 0 \quad (8 \text{ G's})$$

B- Higgs particle. This is a spin-0 particle that is electrically neutral:

$$g_s = 0 \quad h \quad m_h > 114 \text{ GeV}$$

Standard Model

This is the only particle that has not been observed yet. The aim of the Large Hadron Collider (LHC) is to discover the Higgs. The role of the Higgs is to give mass to other

particles in the Standard Model in a consistent way. The Higgs represents one degree of freedom

Calculating  $g_*$  in the Standard Model:

The heaviest particle in the Standard Model is the top quark "t" ( $m_t \sim 170$  GeV). This means that at  $T \gg 170$  GeV, all of the Standard Model particles are in the relativistic regime. Also that they are also in thermal equilibrium (this we will discuss in detail later on), we then find

$$g_* = \sum_{\text{bosons}} g_B + \frac{7}{8} \sum_{\text{fermions}} g_F$$

First lets find  $\sum g_B$ . We have:

$$\sum g_B = 1 + 2 + 2 \times 3 + 2 \times 8 = 28$$

Higgs     $\gamma$              $Z, W^\pm$             gluons

For the fermionic degrees of freedom we have:



$$\sum g_F = \underbrace{3 \times 4 \times 6}_{\text{quarks}} + \underbrace{4 \times 3}_{\text{charged leptons}} + \underbrace{2 \times 3}_{\text{neutrinos}} = 90$$

Thus:

$$g_* = 28 + \frac{7}{8} \times 90 = 106.75$$

This is the number of relativistic degrees of freedom in the universe at temperatures above 170 GeV (which amounts to very early times).

At these temperatures we have:

$$H^2 = \frac{\rho}{3M_p^2} = \frac{\pi^2}{90} \times 106.75 \frac{T^4}{M_p^2} \quad M_p = (8\pi G)^{-\frac{1}{2}} = 2.4 \times 10^{18} \text{ GeV}$$

↓  
reduced Planck mass

$$\Rightarrow H \sim 3 \frac{T^2}{M_p} \quad \leftarrow T \gg 170 \text{ GeV}$$

As the universe expands the temperature decreases.

As it drops below the mass of a particle, the particle decays to lighter particles but the inverse decays

will not be efficient because of the exponential suppression in the density of particles with an energy  $E \gg T$ . The decays are very effective in, and hence the number density of <sup>unstable</sup> particles whose mass is  $\gg T$  essentially goes to zero. Because of the effectiveness of the decay, this is an adiabatic process, and hence the total entropy remains constant while the number of relativistic degrees of freedom decreases.

Now let's turn to the time when  $T \sim 1$  MeV. At this temperature only photons (2 degrees of freedom), electrons and positrons (4 degrees of freedom) and three neutrinos (6 degrees of freedom) are in the relativistic regime and present in the thermal bath. Then,



the age of the universe at some other important moments ( $g_*$  is not the same at all temperatures, hence this is just an order of magnitude estimate)

-  $T \sim 200 \text{ MeV}$ : at this time quarks are bound and form hadrons (quark hadron phase transition).

We find  $t \sim 10^{-5} \text{ sec}$  at this time. This is the epoch that we can reconstruct through heavy ion collisions at RHIC experiment.

-  $T \sim 5 \text{ GeV}$ : in most popular models of particle dark matter the annihilation of dark matter particles annihilate at this time, and comoving number density of dark matter particles remain constant from then on. We find  $t \sim 10^{-4} \text{ sec}$  at this time. If we detect dark matter, and can

measure its mass and couplings at colliders, then we may have an experimental probe of such an early moment in the history of the universe.

$T \sim 100 \text{ GeV}$ ; at this time the electroweak symme<sup>try</sup> breaks down and  $Z, W^\pm$  particles become massive.

This epoch is called "electroweak phase transition" at which time  $t \sim 10^{-10} \text{ sec}$ . We have been able to produce particles at such energies at colliders.

$T \sim 10^{16} \text{ GeV}$ ; if the universe was indeed in a thermal equilibrium phase all the way back to such high temperatures, then the strength of the electromagnetic, weak and strong interactions were the same then.

This epoch is called "Grand unified phase transition" and happens when  $t \sim 10^{-33} \text{ sec}$ .

$T \sim 10^{19}$  GeV; at this time gravity becomes very important and <sup>quantum</sup> effects related to it must be taken into account. A consistent theory of quantum gravity (like string theory) is needed to describe physics then. This is called the "Planck epoch" at which  $t \sim 10^{-44}$  sec (the Planck time).