Lec 11: 02/23/2010

Standard Model of Particle Physics:

In order to calculate g_*, we need to know the number of degrees of freedom that are relativistic at a given temperature and are in thermal equilibrium.

Let us discuss the "Standard Model of Particle Physics" that describes the known fundamental particles and their interactions (except for gravity) and is in very good agreement with experiments.

The fundamental particles come in three groups according to the Standard Model;

1- Building blocks of matter; these are spin-$\frac{1}{2}$ fermions

2- Mediators of the forces; these are spin-1 bosons.

3- Higgs field; it is a spin-0 boson.

We now describe each of these groups in more detai

1- Building blocks of matter come in two types:

(a) Quarks. These are fermions that have all the three (electromagnetic, weak, strong) interactions. The nucleons (protons and neutrons) are bound states of two quarks:

$$\begin{array}{lll} q=+\frac{2}{3} & \left[\begin{array}{l} u \ (\text{up}) \\ d \ (\text{down}) \end{array}\right. & \begin{array}{l} m_u \sim 2\,\text{MeV} \\ m_d \sim 5\,\text{MeV} \end{array} \\ q=-\frac{1}{3} & & \end{array}$$

"q" denotes the electric charge of the quarks. Proton is a uud bound state, while neutron is a udd bound state. The quarks are bound by the strong force.

There are two heavier copies of quarks with the same charge as u, d respectively:

$$\begin{array}{llll} q=+\frac{2}{3} & \begin{bmatrix} c \\ s \end{bmatrix} \begin{array}{l} (\text{charm}) \\ (\text{strange}) \end{array} & \begin{array}{l} m_c \sim 1.3\,\text{GeV} \\ m_s \sim 110\,\text{MeV} \end{array} & \begin{bmatrix} t \\ b \end{bmatrix} \begin{array}{l} (\text{top}) \\ (\text{bottom}) \end{array} \quad \begin{array}{l} m_t \sim 170\,\text{GeV} \\ m_b \sim 4\,\text{GeV} \end{array} \\ q=-\frac{1}{3} & & & \end{array}$$

We do not see free, isolated quarks at low energies because they are bound by the strong force (which becomes strong at low energies).

All quarks come in three colors (namely red, blue and green), which can be considered as quantum charges related to strong interactions (similar to electric charge and electromagnetic interaction. Also note that quarks are spin-$\frac{1}{2}$ fermions. This represents four degrees of freedom (spin-up and spin-down particle and antiparticle).

(b) Leptons. These are fermions that have only weak and electromagnetic interactions. The leptons are either charged ($q = -1$) or neutral ($q = 0$). The lightest charged lepton is the electron that binds to nuclei via electromagnetic interactions and form atoms. The leptons also come in three copies (called family or flavor

$$\begin{array}{lccccc} q=0 & \nu_e & & \nu_\mu & & \nu_\tau \\ q=-1 & e & m_e \sim 0.5\ \text{MeV} & \mu & m_\mu \sim 105\ \text{MeV} & \tau \quad m_\tau \sim 1.7\ \text{GeV} \\ & \downarrow & & \downarrow & & \downarrow \\ & \text{electron} & & \text{muon} & & \text{tau} \end{array}$$

The neutrinos associated with the charged leptons are called electron neutrino, moon neutrino and tau neotrino.

From atmospheric neutrino oscillations, there exists a mass scale of ~ 0.05 eV for the difference of neutrino masses. The combined limits from lab and cosmology puts an upper bound $\lesssim 0.50$ eV on the sum of the mass of three neutrinos.

The charged leptons each represent four degrees of freedom. But each neutrino represents two degrees of freedom. This is what nature seems to have dictated. A naive (but not quite correct) way to understand it is that since neutrinos are neutral particles, then particle and antiparticle can be the same (unlike the case of quarks or charged leptons). This will leave us with spin-up and spin-down states, that count for two degrees of freedom.

2 - Mediators of force are spin-1 particles. The most famous one is photon (γ) that is the messanger of electromagnetic interactions:

$q=0$ γ $m_\gamma = 0$

Since photon is massless ($m_\gamma = 0$ compatible with all experimental bounds), then it has two degrees of freedom (the transverse polarizations).

Next we have three spin-1 particles that are messangers of the weak interactions:

$q=0$ $\quad Z \quad m_Z \sim 90$ GeV

$q=\pm 1$ $\quad W^\pm \quad m_{W^\pm} \sim 80$ GeV

The fact that $Z, W^\pm$ are massive explains why the weak interactions are short range; the exchang of $Z, W^\pm$ gives rise to Yukawa potentials (whic are exponentially suppressed in distance) unlike the photon exchange that results in a Coulomb potential with infinite range.

Since $Z, W^\pm$ (called weak gauge bosons) are massi each of the represents three degrees of freedom: two transverse and one longitudinal polarizations.

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Finally, we have the messangers of strong interac that are called gluons (denoted by "G"). There are 8 gluons, and they are massless. Hence each gluon represents two degrees of freedom (like the photon). However, at low energies we do not see free, isolated gluons because they are bound by strong interactions:

$q=0$ G $m_G=0$ (8 G's)

3. Higgs particle. This is a spin-0 particle that is electrically neutral:

$q=0$ h $m_h > 114$ GeV

This is the only Standard Model particle that has not been observed yet. The aim of the Large Hadron Collider (LHC) is to discover the Higgs. The role of the Higgs is to give mass to other particles in the Standard Model in a consistent way. The Higgs represents one degree of freedom

Calculating g_* in the Standard Model:

The heaviest particle in the Standard Model is the top quark "t" ($m_t \sim 170$ GeV). This means that at $T \gg 170$ GeV, all of the Standard Model particles are in the relativistic regime. Assuming that they are also in thermal equilibrium (this we will discuss in detail later on), we the find,

$$g_* = \sum_{bosons} g_B + \frac{7}{8} \sum_{fermions} g_F$$

First lets find $\sum g_B$. We have:

$$\boxed{\sum g_B = \underset{\text{Higgs}}{1} + \underset{\gamma}{2} + \underset{Z,\,W^\pm}{2 \times 3} + \underset{\text{gluons}}{2 \times 8} = 28}$$

For the fermionic degrees of freedom we have:

$$\sum g_F = \underset{\underset{\text{quarks}}{\downarrow}}{3\times 4\times 6} + \underset{\underset{\text{charged leptons}}{\downarrow}}{4\times 3} + \underset{\underset{\text{neutrinos}}{\downarrow}}{2\times 3} = \boxed{90}$$

Thus:

$$g_* = 28 + \frac{7}{8}\times 90 = \boxed{106.75}$$

This is the number of relativistic degrees of freedom in the universe at temperatures above 170 GeV (which amounts to very early times).

At these temperatures we have,

$$H^2 = \frac{\rho}{3M_p^2} = \frac{\pi^2}{90}\times 106.75\,\frac{T^4}{M_p^2} \qquad \underset{\underset{\text{reduced Planck mass}}{\downarrow}}{M_p} \equiv (8\pi G)^{-\frac{1}{2}} = 2.4\times 10^{18}\,\text{GeV}$$

$$\Rightarrow \boxed{H \sim 3\,\frac{T^2}{M_p}} \longleftarrow T \gg 170\,\text{GeV}$$

As the universe expands the temperature decreases. As it drops below the mass of a particle, the particle decays to lighter particles but the inverse decays will not be efficient because of the exponential suppression in the density of particles with an energy $E \gg T$. The decays are very effective in, and hence the number density of unstable particles whose mass is $\gg T$ essentially goes to zero. Because of the effectiveness of the decay, this is an adiabatic process, and hence the total entropy remains constant while the number of relativistic degrees of freedom decreases.

Now lets turn to the time when $T \sim 1$ MeV. At this temperature only photons (2 degrees of freedom), electrons and positrons (4 degrees of freedom) and three neutrinos (6 degrees of freedom) are in the relativistic regime and present in the therma bath. Then:

$$g_* = \underset{\gamma}{2} + \frac{7}{8}(\underset{e^{\pm}}{4} + \underset{\substack{\nu_e \\ \nu_\mu \\ \nu_\tau}}{6}) = \boxed{10.75}$$

As a result,

$$H^2 = \frac{\rho}{3M_p^2} = \frac{\pi^2}{90} \times 10.75 \frac{T^4}{M_p^2} \Rightarrow H \sim \frac{T^2}{M_p}$$

In a radiation dominate universe we have $H = \frac{1}{2t}$,

which results in;

$$\boxed{t \sim \frac{M_p}{T^2}} \xleftarrow{\quad} T \sim 1\,\text{MeV}$$

For $T \sim 1$ MeV, we find $t \sim 1$ sec. Therefore, the universe was about a second old when the temperature was ~ 1 MeV. As we will see later, this is a very important moment as the weak interactions freeze out at this time and drop out of equilibrium. Th represents the onset of big-bang nucleosynthesis.

Using the relationship $t \sim \frac{M_p}{T^2}$, we can find the age of the universe at some other important moments (g_* is not the same at all temperatures, hence this is just an order of magnitude estimate;

- $T \sim 200$ MeV: at this time quarks are bound and form hadrons (quark hadron phase transition). We find $\underline{\underline{t \sim 10^{-5}\ \text{sec}}}$ at this time. This is the epoch that we can reconstruct through heavy ion collisions at RHIC experiment.

- $T \sim 5$ GeV: in most popular models of particle dark matter the annihilation of dark matter particles annihilate at this time, and comoving number density of dark matter particles remain constant from then on. We find $\underline{\underline{t \sim 10^{-7}\ \text{sec}}}$ at this time. If we detect dark matter, and can measure its mass and couplings at colliders, then we may have an experimental probe of such an early moment in the history of the universe.

- $T \sim 100$ GeV: at this time the electroweak symmetry breaks down and $Z, W^{\pm}$ particles become massive. This epoch is called "electroweak phase transition", at which time $\underline{\underline{t \sim 10^{-10}\ \text{sec}}}$. We have been able to produce particles at such energies at colliders.

- $T \sim 10^{16}$ GeV: if the universe was indeed in a therma equilibrium phase all the way back to such high temperatures, then the strength of the electromagneti weak and strong interactions were the same then. This epoch is called "Grand unified phase transition" and happens when $t \sim 10^{-33}$ sec.

$T \sim 10^{19}$ GeV; at this time gravity becomes very important and quantum effects related to it must be taken into account. A consistent theory of quantum gravity (like string theory) is needed to describe physics then. This is called the "Planck epoch" at which $t \sim 10^{-44}$ sec (the Planck time).